Ion distribution function and radial profile of neutron production rate in spherical inertial electrostatic confinement plasmas

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Abstract. The radial profile of the neutron production rate in spherical inertial electrostatic confinement plasmas is investigated. The electrostatic potential is obtained by solving the Poisson equation, and by using the potential; the fuel ion velocity distribution function is determined at each radial point. From the velocity distribution function, the neutron production rate is accurately evaluated. Numerical results show that if it is assumed that fuel ions are confined keeping the total energy and angular momentum almost constant, the double radial peak in the neutron production rate can appear without creation of the deep double potential well.

1. Introduction

Spherical inertial electrostatic confinement (SIEC) is a concept for electrostatically confining high energy ions in the potential wells produced by the circulating ions themselves. In spherical devices, ions are accelerated towards the centre of the sphere, and are decelerated again outwards in the interelectrode space. Fusion reactions in SIEC devices were first realized by Hirsh [1, 2] using high energy ion guns as a source of ions. He observed the double radial peak of the neutron production rate as well as a T(d,n)^{4}He (D(d,n)^{3}He) neutron generation rate of more than 10^9 (10^6) n/s. The results seem to suggest the creation of a double potential well; a double peak of the neutron production rate may be formed owing to the double density peak that is induced by the double potential well. However, the measured neutron production rate was proportional to the grid current, and then the possibility that neutron production was sustained mainly by fusion reactions between accelerated ions would be quite low. Recently, Miley et al. [3, 4] carried out experiments using a transparent grid as cathode and a spherical glow discharge as ion source. In their experiments, although the measured neutron production rate was a fraction [5] of that of Hirsh's data, the dependence of the neutron production rate on the device parameters was similar to that of Hirsh. They explained the current, pressure and external voltage dependences of the neutron production rate by assuming that the fusion reactions occur between accelerated ions and background neutral particles [3, 5, 6]. The assumption that neutron production is sustained mainly by fusion reactions between accelerated ions and background neutral particles implies that the effect of the double potential well in SIEC fusion is weak. The reason why the double peak in the radial profile of the neutron production rate appears in the case of beam–background (target) fusion should be clarified.

In this Letter, we consider a deuterium plasma confined in an SIEC device and focus our attention on the radial profile of the neutron production rate. By solving the Poisson equation for fixed ion and electron distribution functions, we calculate the electrostatic potential. From the potential obtained we derive the ion velocity distribution function and accurately evaluate the D(d,n)^{3}He neutron production rate at each radial point. A possible mechanism for the appearance of a double peak in the neutron production rate is discussed.

2. Analysis model

The SIEC is a weak collisional system, where the collision frequency and fusion reaction rate are small compared with the transit frequency of charged particles in a given electrostatic potential field. The motion of a charged particle in such a system is described by two constants of motion: the total energy $E = \frac{1}{2}mv^2 + q\phi$ and the angular momentum $L = mrv_\perp$. In this Letter, we assume that the deuterons are confined, keeping the quantities E and L almost constant.

The ion and electron distribution functions in the above system can be expressed by an arbitrary function with respect to E and L. In previous work, there have been attempts to express their shapes, for example [7], by using δ functions and/or Heaviside functions. In this case, however, the radial profiles of potential and density tend to have unnatural shapes including discontinuities. This could be attributed to the peculiarities of these functions. To avoid this problem, we simulate the distributions using an exponential function, so that their shapes are close to those expressed by a Heaviside function. Electrons are assumed to be generated with low kinetic energy at the cathode. Weak Coulomb collisions may slowly relax the distributions towards a Maxwellian. The timescale on which the distribution broadens would be larger towards the energy direction than towards the angular momentum direction. Hence in this Letter, using the dimensionless parameters α and β_a, we assume the distributions to be of the following form:

$$f_a(E,L) = c_a \exp\left[-\left(\frac{E-|q_a\phi_0|}{\alpha E_0}\right)^2 - \left(\frac{L}{\beta_a L_0}\right)^2\right] \quad (1)$$

where the subscript a represents particle species, i.e. deuteron or electron, grid voltage ϕ_0, $E_0 = 10$ keV and $L_0 = r_{cat}\sqrt{2m_D E_0}$. By adjusting the α and β_a values, we can simulate the broadness of the distributions towards the energy and angular momentum directions. The coefficient c_a is determined so that the deuteron (electron) density at the cathode $n_i(r_{cat})$ ($n_e(r_{cat})$) is equal to n_i^{cat} (n_e^{cat}). In this Letter, following the treatment of Thorson et al. [5, 8], we relate n_i^{cat} to the measured cathode current I_{meas} by

$$n_i^{cat} = \frac{1}{1-\gamma^2}\frac{1}{1+\delta}\frac{I_{meas}}{4\pi r_{cat}^2\sqrt{2e\phi_0/m_D}e} \quad (2)$$

where γ represents the transparency factor of the inner grid [2] and δ is the number of secondary electrons emitted from the grid due to ion impact [8]. Throughout the calculation, we assume that $\gamma = 0.95$ and $\delta = 1$. We consider the secondary electrons which are drawn inside the cathode with low kinetic energy, for example, several electronvolts. These electrons would pass through the centre only once. When $\phi_0 \approx 10$–15 kV, by subtracting the effect of the increase in ion current owing to the circulation from Eq. (2), we assume the electron density at the cathode to be $n_e^{cat} = (1-\gamma^2)n_i^{cat}$.

The electrostatic potential structure can be determined by the Poisson equation,

$$\nabla^2\phi(r) = -\frac{q_i n_i(r) - q_e n_e(r)}{\varepsilon_0}. \quad (3)$$

By means of the computational iterative method, we solve Eq. (3) for fixed distribution functions, i.e. Eq. (1), and obtain the structure of the radial potential. From the potential obtained, the deuteron velocity distribution function $F_D(r, v_\parallel, v_\perp)$ and the deuteron density are determined at each radial point. Here $v_\parallel$ represents a radial velocity component and $v_\perp$ the vertical component in spherical co-ordinates. By using the velocity distribution function, we can evaluate the D(d,n)^{3}He fusion reaction rate coefficient, and by multiplying the reaction rate coefficient by the deuteron and background deuterium densities at each radial point, the radial profile of the neutron production rate owing to fusion reactions between deuterons and neutral deuterium gas can be estimated.

Throughout the calculations, the fusion cross-sections are taken from the work of Duane [9] who assumed fusion reactions between bare nuclei.

3. Results and discussion

In Fig. 1 we first show the radial profiles of both (a) the potential and (b) the deuteron and electron densities for fixed ion and electron distribution functions ($\alpha = 0.04$, $\beta_D = 0.08$ and $\beta_e = 0.0005$). The grid voltage is taken as $\phi_0 = 15$ kV, cathode current as $I_{meas} = 80$ mA and cathode radius as $r_{cat} = 3$ cm. In this Letter we do not chose a strongly focused distribution for electrons (even if they have the same angular momentum spread as that of the ion distribution and β_e becomes smaller than β_D owing to the mass ratio), and hence the potential almost has a single peak at the centre of the sphere. From the potential and deuteron densities, the velocity distribution function $F_D(r, v_\parallel, v_\perp)$ can be estimated at each radial point. In Fig. 2 the two dimensional deuteron velocity distribution functions at (a) $r = 0.05$ cm, (b) $r = 1$ cm and (c) $r = 2$ cm are exhibited. The distribution functions and variables are normalized using v_0, the thermal speed for 15 keV deuterons. It is found that the velocity distribution function becomes isotropic near the central core region. This is because, when the ions are assumed to move keeping with their angular momentum constant then their vertical velocity component $v_\perp$ becomes larger as they approach the centre of the sphere. Furthermore, it should be noted that the peak of the velocity distribution is shifted to a low kinetic energy range at small radius. This implies that the ions are decelerated near the central region, the reason being the rapid increment in the potential around the central core region (Fig. 1).

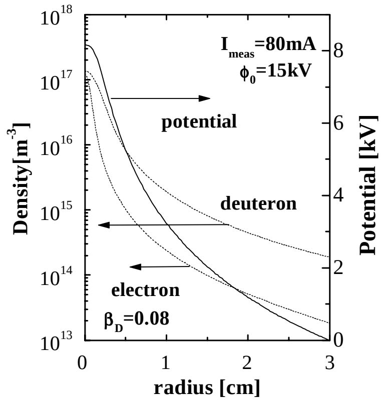

Figure 1. Radial profiles of deuteron and electron densities, and that of the potential.

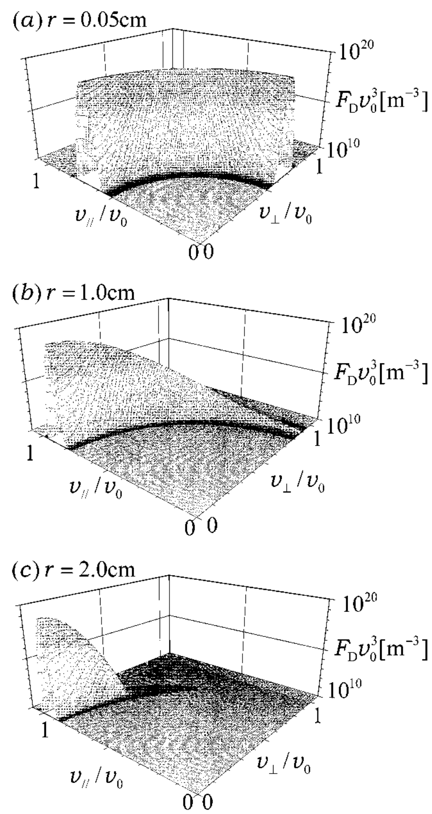

Figure 2. Deuteron velocity distribution function at (a) $r = 0.05$ cm, (b) $r = 1$ cm and (c) $r = 2$ cm.

Using the deuteron velocity distribution function obtained, the D(d,n)^{3}He fusion reaction rate coefficient between accelerated deuterons and neutral background deuterium gas is calculated, being shown in Fig. 3 as a function of radius. Here the background deuterium gas is assumed to have a Maxwellian distribution of temperature 0.1 eV, and the density is taken as 10^{20} m^{-3}. Both the relaxation of the

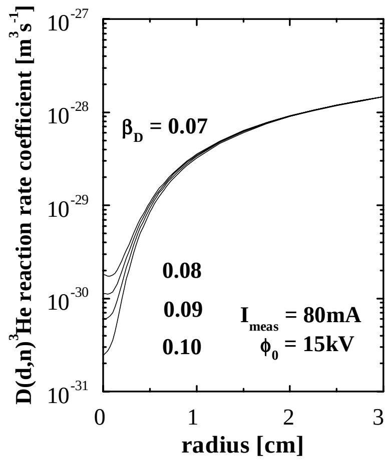

Figure 3. Radial profile of the D(d,n)^{3}He fusion reaction rate coefficient for various β_D values.

velocity distribution function towards an isotropic form and the deceleration of ion speed cause a reduction of the fusion reaction rate coefficient. We found that the reaction rate coefficient decreases gradually with decreasing radius around the cathode region and decreases rapidly with decreasing radius near the central core. Multiplying the reaction rate coefficient by the accelerated deuteron and background deuterium densities, the D(d,n)^{3}He neutron production rate is evaluated, with the result being shown in Fig. 4. For small β_D values, the central peak of the deuteron density becomes large, while the reaction rate coefficient decreases more rapidly with decreasing radius owing to the increment in the potential. The neutron production rate then has small values over the entire radial range, and its radial peak moves outwards from the centre ($r = 0$). On the contrary, with increasing β_D values, the absolute values of the neutron production rate become large and the double peak begins to appear. The total neutron production rate is examined by integrating the neutron production rate inside the cathode, i.e. $r \leq 3$ cm. When we assume that $\alpha = 0.04$, $\beta_D = 0.08$, $I_{meas} = 90$ mA and $\phi_0 = 15$ kV, the D(d,n)^{3}He neutron production rate caused by the fusion reactions between accelerated deuterons and background deuterium gas is estimated to be 4.3×10^2 n/s.

In this Letter, we have chosen adequate β_D and β_e values to numerically reproduce the double peak in the radial profile of the neutron production rate, and have shown a possible mechanism for the appearance of the double peak. The position and height of the double neutron peak are influenced by the grid current and voltage, the background deuterium

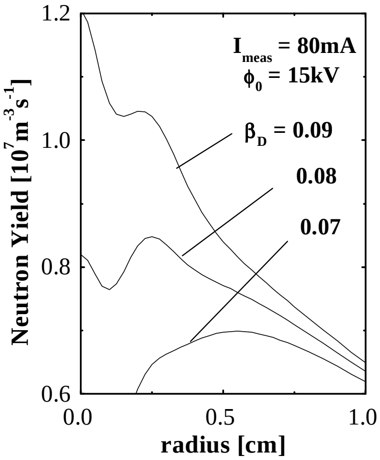

Figure 4. Radial profile of the D(d,n)^{3}He reaction rate (neutron production rate) for various β_D values ($\phi_0 = 15$ kV and $I_{meas} = 80$ mA).

temperature and the cathode ion to electron density ratio. In order to verify the mechanism, it would be necessary to identify the ion and electron distributions in experiments. In any case, however, the fusion reaction rate coefficient decreases around the central core, while, on the contrary, the fuel ion density increases towards the centre of the sphere. It can be said that SIEC has an intrinsic mechanism to produce the double peak in neutron production rate even if a deep double potential well (double density peak) is not created.

We have chosen calculation conditions where a double potential well is not easily generated. In this treatment, however, we do not intend to deny the appearance of the double potential well. (If we assume a higher concentration for electrons, a double potential well would be created.) The potential structure is essentially determined by the balance between ion and electron densities at each radial point. Electrons move in the device faster than ions, and an unstable double well may be generated with fluctuations as predicted by Ohnishi et al. [10]. The effect of a double potential well on both the neutron production rate and its radial profile, and the way in which the neutron production rate itself depends on the grid current will be discussed in a subsequent article, including the contribution of the fusion reactions between accelerated deuterons.

In this Letter we have expressed the ion and electron distribution functions by using an exponential function, i.e. Eq. (1). The results are influenced by the broadness of the distributions towards the angular momentum direction, while similar results would be obtained if we adopt other functions, for example, a δ function and/or a Heaviside function, to express the ion and electron distributions. We have assumed weak collisional plasmas, and then we have assumed nearly monoenergetic distributions, i.e. α is fixed at 0.04. (In this Letter, the electrons are assumed to be generated with low kinetic energies at the cathode.) If the collisional processes becomes more important, the distribution may be broadened, and the charge exchange reaction may cause an increase in the temperature (and/or high energy component) of the background neutral gas [5]. The influence of the α values on the radial profile of the potential and densities is small, while the collisional process may increase the neutron production rate. In this case, Eq. (1) may also underestimate the reduction of the fusion reaction rate coefficient around the central core region (a double peak of the radial neutron production rate may appear for a smaller cathode current than that of our calculation).

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