Modification of alpha-particle emission spectrum in beam-injected deuterium-tritium plasmas

H. Matsuura and Y. Nakao
Department of Applied Quantum Physics and Nuclear Engineering, Kyushu University, Motooaka, Fukuoka 819-0395, Japan

(Received 11 November 2008; accepted 6 March 2009; published online 14 April 2009)

The alpha (\(\alpha\))-particle and neutron emission spectra in a deuterium-tritium plasma accompanied with neutral-beam-injection (NBI) heating are evaluated in a consistent way by solving the Boltzmann–Fokker–Planck equations for deuteron, triton, and \(\alpha\)-particle simultaneously. It is shown that owing to the existence of non-Maxwellian tail component in fuel-ion distribution function due to NBI and/or nuclear elastic scattering, the generation rate of the energetic (\(\geq 4\) MeV) \(\alpha\)-particle increases significantly. When 20 MW intense deuterium beam with 1 MeV beam-injection energy is injected into an 800 m\(^3\) plasma (\(T_e=10\) keV, \(n_e=6.2 \times 10^{19}\) m\(^{-3}\)), the enhancement of the fraction of the power carried by \(\alpha\)-particles with energy above 4 (3.9) MeV to total \(\alpha\)-particle power is almost twice (1.5 times) as much from the value for Gaussian distribution. A verification scenario for the modification of the emission spectrum by using the gamma (\(\gamma\))-ray-generating \(^7\)Be(\(\alpha,n\gamma\))\(^{12}\)C reaction is also presented. © 2009 American Institute of Physics.

[DOI: 10.1063/1.3106683]

I. INTRODUCTION

Energetic ions in a burning plasma have important roles in various stages of fusion-reactor operations. Nuclear elastic scattering\(^1,2\) (NES) of fusion-produced \(\alpha\)-particle by thermal ion contributes to the energetic knock-on tail formation.\(^3,13\) In a deuterium-tritium (DT) plasma, the resulting modification of the neutron emission spectrum was computed,\(^5\) and the knock-on tail formation in fuel-ion velocity distribution function was experimentally ascertained\(^6,7\) by observing the deviation of the neutron emission spectrum from Gaussian distribution.\(^6\) Aiming at plasma diagnostics, an analytical technique to accurately specify the spectrum of fusion product in Maxwellian plasma has been proposed.\(^8\) It is also well known that the knock-on tail is created by energetic ions produced by neutral-beam-injection\(^9,10\) (NBI) and/or ion cyclotron range of frequency\(^11\) heating. We have shown that when NBI heating is applied, more significant knock-on tail is formed,\(^9,10\) and NES effects on fractional beam-energy deposition to ions\(^8,12\) and T\((d,n)\)^{\(^3\)He} reaction rate coefficient\(^10\) are enhanced to some extent. In such a beam-injected plasma the modification of the emission spectrum would be further conspicuous and a significant modification in \(\alpha\)-particle emission spectrum would be observed.

The nonthermal fusion has been examined for a number of decades\(^14\) and suitable models to predict the nonthermal fusion yields have been developed. In the DT burning experiment,\(^15\) simulation using TRANSP (Ref. 16) successfully explained the \(\alpha\)-heating performance. In conceptual designs of next-generation fusion devices, however, use of deuterium beam energy much higher than the current experiments can be considered. In such a case, the effects of modification of fusion-product emission spectrum as well as NES between beam deuteron and background ions may be appreciable. Although it has been known that a modification of the emission spectrum can appear in \(\alpha\)-particle spectrum, it is the fact that many previous simulations concerning to the DT burning plasma have been performed assuming Gaussian distribution (or delta function) for the \(\alpha\)-particle emission spectrum. Undoubtedly if the non-Maxwellian component in the fuel-ion velocity distribution function is small, the influences of the broadness of the \(\alpha\)-particle emission spectrum would be negligible. However, how about when an intense neutral beam is injected and a significant non-Maxwell component is created? In future fusion reactors, external heating may be continuously adopted even in a steady-state operation. The transport processes, e.g., first-orbit loss, of \(\alpha\)-particle in the fusion devices would be affected by the particle energy. The fractional energy deposition from \(\alpha\)-particle to bulk ions and electrons via collisions is also changed depending on the relative velocity between \(\alpha\) and bulk particles. The modification of the \(\alpha\)-particle emission spectrum, e.g., increment in the number of energetic (\(\geq 3.5\) MeV) \(\alpha\)-particle, may influence the \(\alpha\)-heating characteristics to some extent. In the same manner, the modification of neutron emission spectrum may influence the neutronics in the reactor blanket system and the load to the surrounding materials. It is hence important to grasp the degree of the modifications of \(\alpha\)-particle and neutron emission spectrum in a beam-injected plasma quantitatively.

In this paper, we consider a DT plasma accompanied with injection of a monoenergetic deuterium beam. On the basis of the Boltzmann–Fokker–Planck (BFP) model,\(^9,10,13\) the modifications of the \(\alpha\)-particle and neutron emission spectra are evaluated simultaneously considering the distortion of deuteron, triton and \(\alpha\)-particle distribution functions. The use of the \(\gamma\)-ray-generating \(^7\)Be(\(\alpha,n\gamma\))\(^{12}\)C reaction as a plasma diagnostic tool has been proposed by Kiptily.\(^17,18\)

---

\(^{9,10,13}\)Electronic mail: mat@nucl.kyushu-u.ac.jp.
One of the $\gamma$-ray-generating branches of the $^9\text{Be}(\alpha,n\gamma)^{12}\text{C}$ reaction

$$\alpha + ^9\text{Be} \rightarrow ^{12}\text{C}^*[7.65] + n, \quad ^{12}\text{C}^*[7.65] \rightarrow ^{12}\text{C}^*[4.44] + \gamma \quad (E_\gamma = 3.21 \text{ MeV})$$

could be adopted to verify the modification of the $\alpha$-particle emission spectrum. A scenario to verify the modification in various beam-injected plasmas using the $\gamma$-ray-generating $^9\text{Be}(\alpha,n\gamma)^{12}\text{C}$ reaction is discussed.

II. ANALYSIS MODEL

The ion velocity distribution function in burning plasma can be estimated by solving the following BFP equation for ion species $i$ ($i = \text{D, T, and } \alpha$-particle),

$$\sum_j \left( \frac{\partial f_i}{\partial t} \right)_j + \sum_k \left( \frac{\partial f_i}{\partial v} \right)_{\text{NES}} + \frac{1}{v^2} \frac{\partial}{\partial v} \left( \frac{v^2 f_i}{2 m_i} \right) + S_i(v) - L_i(v) = 0,$$  \hspace{1cm} (1)

where $f_i(v)$ is the velocity distribution function of the species $i$. The first term in the left-hand side of Eq. (1) represents the effect of the Coulomb collision. The summation is taken over all background species, i.e., $j = \text{D, T, } \alpha$-particle and electron. The collision term is hence nonlinear, retaining collisions between ions of the same species. The second term accounts for the NES of species $i$ by background ion species $k$.\(^9,13\) We consider NES between $\alpha$-particle and D, and between $\alpha$ and T, i.e., $(i,k) = (\text{D, } \alpha), (\text{T, } \alpha), (\alpha, \text{D}),$ and $(\alpha, \text{T}).$ The NES cross-sections are taken from the work of Perkins and Cullen.\(^2\)

The third term in the left-hand side of Eq. (1) represents the diffusion in velocity space due to thermal conduction. To incorporate the unknown loss mechanism of energetic ions into the analysis, we simulate the velocity dependence of the energy loss due to thermal conduction and the particle-loss time.\(^9,13\) The source $[S_i(v)]$ and loss $[L_i(v)]$ terms take different form for every ion species. For deuteron, the source and loss terms are described so that the fueling, beam injection, transport loss, and the loss due to $\text{T}(d,n)^4\text{He}$ reaction are balancing.\(^9,13\) The NBI rate per unit volume $S_{\text{NBI}}$ is expressed using the beam-energy $E_{\text{NBI}}$ and injection power $P_{\text{NBI}},$ i.e., $S_{\text{NBI}} = P_{\text{NBI}} / (E_{\text{NBI}} V).$ Here $V$ represents the plasma volume and $V = 800 \text{ m}^3$ is assumed.\(^19\) For triton the NBI injection term has not been included, and the source and loss terms are described so that the fueling rate, transport loss, and the loss due to $\text{T}(d,n)^4\text{He}$ reaction are balancing.\(^10\)

For $\alpha$-particle, the source term due to $\text{T}(d,n)^4\text{He}$ reaction is written as

$$S_\alpha(v) = \frac{(dE/db) \cdot N_\alpha}{4 \pi v^2} \frac{dN_\alpha}{dE}.$$ \hspace{1cm} (2)

Here $N_\alpha(E)$ represents the $\alpha$-particle generation rate. Its energy spectrum is described as

![FIG. 1. (Color online) Deuteron distribution functions for several beam-injection powers. The electron temperature $T_e = 20 \text{ keV}$, energy and particle confinement times $\tau_p = 3$ s, and beam-injection energy $E_{\text{NBI}} = 1 \text{ MeV}$ are assumed.](image-url)

\[\text{Deuteron Energy [keV]}\]

\[\text{Distribution Functions}\]

\[\text{Maxwellian}\]

\[\text{w/o NBI}\]

\[\text{FIG. 1. (Color online) Deuteron distribution functions for several beam-injection powers. The electron temperature $T_e = 20 \text{ keV}$, energy and particle confinement times $\tau_p = 3$ s, and beam-injection energy $E_{\text{NBI}} = 1 \text{ MeV}$ are assumed.}\]
Modification of alpha-particle emission function. The relative intensity of the tail component is formed below 1 MeV energy range in the deuteron distribution function. It is found that the non-Maxwellian tail due to NBI knock-on tail due to NES of a Gaussian distribution. The spectrum when no NBI is made and is $P_{\text{NBI}}=40$ MW are assumed.

The neutron emission spectrum, normalized to the total generation rate, in deuterium-tritium plasmas is exhibited in Fig. 2 as a function of neutron energy in the laboratory system. The calculation conditions are the same as those in Fig. 1. The bold line expresses the neutron emission spectrum when no NBI heating is applied. The dotted line denotes a Gaussian distribution corresponding to the case when both NBI and NES are not considered. The neutron emission spectrum is broadened toward both low and high energy regions, and the intensity of the low and high energy tails in the spectrum increases with increasing NBI powers. The normalized $\alpha$-particle emission spectra for several beam-injection energies are also presented in Fig. 3. The NBI heating power is taken as 40 MW, and other parameters are the same as those in Figs. 1 and 2. The broadness of the $\alpha$-particle emission spectrum toward low and high energy ranges is also conspicuous, which becomes significant with increasing beam-injection energy. For example, when NBI heating with 800 keV deuterium beam energy is made, the fraction of the generation rate of $\alpha$-particle with 5 MeV birth energy is $\sim$10 times larger than in the case that no NBI injection is made and is $\sim$100 times larger than the value for Gaussian distribution. The spectrum when no NBI is made (bold line) almost agrees with the previous calculation and experiment. In Ref. 5 the amplitude of the neutron yield $Y'$ from the region $E_n > 15.5$ MeV to total neutron yield $Y$ was evaluated as $Y'/Y \approx 1/5000$ when $T_e (= T_{\text{ion}}) = 20$ keV and $n_D=n_T=(1/2)n_e=5 \times 10^{19}$ m$^{-3}$. We assumed parameters similar to those in Ref. 5 as much as possible, and evaluated the amplitude of the neutron yield from the region $E_n > 15.5$ MeV. When $T_e = 20$ keV, $n_D=n_T=5 \times 10^{19}$ m$^{-3}$, and $n_e = 1.2 \times 10^{20}$ m$^{-3}$ and without NBI heating, the amplitude $Y'/Y$ is evaluated as $\sim 1/5000$. Our model somewhat underestimate the fraction. In Ref. 5 the ion distribution function is divided into two parts, i.e., bulk and tail component, and bulk distribution is assumed to be Maxwellian at the common temperature with electron. In the present model ion temperature is given a little bit higher than that of electron, i.e., 20.9 keV, and electron density is about 20% higher compared with the one in Ref. 5. The increased electron density intensifies the slowing down of energetic ions, and thus the production of energetic (>15.5 MeV) neutron would become relatively smaller compared to the one in Ref. 5. In addition, the different cross section may also play a role in the difference.

As a result of the increment in the energetic component of deuteron distribution function, the fraction of the energetic $\alpha$-particle generation rate to total rate also increases. The spectrums obtained are quantitatively compared to the Gaussian distribution functions. The Gaussian distribution can be written as

$$S_{\alpha}^\text{Gauss}(v) = \frac{m_\alpha E_{\alpha}^0}{4\pi \Delta v} \exp(- (m_\alpha v^2/2 - E_{\alpha}^0)^2/\Delta^2),$$

where

$$\Delta = \sqrt{\frac{4m_\alpha E_{\alpha}^0}{m_n + m_p}}.$$

Here $E_{\alpha}^0 = [m_\alpha/(m_n + m_p)]Q$, $S_{\alpha}^0 = n_D n_T (\sigma_v) v_{\text{tr}}(\text{triton}) n_\text{H}$, and $T_i$ is the averaged bulk-ion temperature, which is defined as $T_i = (n_D T^D_{\text{bulk}} + n_T T^T_{\text{bulk}})/(n_D + n_T)$. The bulk temperature of deuteron $T^D_{\text{bulk}}$ (triton $T^T_{\text{bulk}}$) is determined by comparing the bulk component of the obtained deuteron (triton) distribution function with Maxwellian by mean of the least-squares fitting. The correlation between beam-injection power $P_{\text{NBI}}$ and
ion temperature is shown in Fig. 4 for several beam-injection energies \( E_{\text{NBI}} \) when electron temperature is 10 keV. As was described, ion temperature slightly increases with increasing \( P_{\text{NBI}} \). It is also found that the ion temperature decreases with increasing \( E_{\text{NBI}} \). This is owing to the following facts: (i) fractional power deposition from beam-deuteron to bulk ion (electron) decreases (increases) with increasing \( E_{\text{NBI}} \), (ii) since \( T(d,n)^4\text{He} \) cross section has a peak at \( \sim 100 \text{ keV} \) deuteron energy in the laboratory system, nonthermal fusion rate decreases for higher \( E_{\text{NBI}} \) and (iii) for the same \( P_{\text{NBI}} \) values, the number of injected beam ion has small values for high \( E_{\text{NBI}} \). The Gaussian distribution function obtained by the above procedure, i.e., Eq. (5), is well agreed with the calculated spectrum, i.e., Eq. (2), around peak-energy (\( \sim 3.5 \text{ MeV} \) for \( \alpha \)-particle and \( \sim 14 \text{ MeV} \) for neutron) region (see Figs. 2, 3, and 5) for various plasma conditions.

The \( \alpha \)-particle emission spectrum when \( T_e=10 \text{ keV} \), \( n_e=6.2 \times 10^{19} \text{ m}^{-3} \), \( P_{\text{NBI}}=20 \text{ MW} \), and \( E_{\text{NBI}}=1 \text{ MeV} \) is illustrated in Fig. 5. The dotted line represents Gaussian distribution function obtained from Eq. (5) and solid line denotes the spectrum produced by non-Maxwellian components in deuteron and triton distribution functions, which is defined as the difference between the total spectrum and the Gaussian. Since both of the total and Gaussian spectra have the same production rate, the solid line has negative value around peak, i.e., \( \sim 3.5 \text{ MeV} \) energy region. We can see that the spectrum around \( 4-4.5 \text{ MeV} \) has larger value than that in 2.5-3 MeV. This is because the kinetic energy carried by deuteron and triton before \( T(d,n)^4\text{He} \) reaction occurs is transferred to the kinetic energy of fusion-produced \( \alpha \) and neutron. An alternative approach to analyze the spectrum would have been to separate the emission spectrum in two components: a Gaussian and a non-Gaussian distribution. In such an analysis, the bulk component in ion distribution function is usually assumed to be Maxwellian (the distortion of the bulk deuteron distribution is neglected). In such a treatment the \( \alpha \)-particle spectrum may be inaccurate in the intermediate (\( 4-5 \text{ MeV} \)) energy region.

As was seen in Fig. 5 the difference between Gaussian and spectrum obtained by solving BFP equations becomes appreciable around 4 MeV energy range. To estimate the enhancement of the power fraction carried by energetic (\( \geq 4 \text{ MeV} \)) \( \alpha \)-particles, the following enhancement parameter is introduced:

$$F_{>4 \text{ MeV}} = \frac{\int_{4 \text{ MeV}}^{\infty} E \left( \frac{dN_{\alpha}}{dE} \right) dE}{\int_{0}^{\infty} E \left( \frac{dN_{\alpha}}{dE} \right) dE}.$$

The enhancement parameter when the modification of \( \alpha \)-emission spectrum is considered (\( F_{>4 \text{ MeV}} \)) and the one when Gauss distribution is assumed (\( F_{>4 \text{ MeV}}^{\text{Gauss}} \)) are evaluated, and the increment in the fraction due to the modification, i.e., \( F_{>4 \text{ MeV}}^{\text{Gauss}}/F_{>4 \text{ MeV}} \), is presented in Fig. 6(a) as a function of the NBI heating power for several electron temperatures. It is found that the increment is more significant at lower electron temperature. This is because in low-temperature range, the half width of the Gaussian distribution decreases, thus the modification of the emission spectrum from the Gaussian distribution becomes more conspicuous. The absolute values for the fraction of the power carried by energetic (\( \geq 4 \text{ MeV} \)) \( \alpha \)-particles when the modification is considered (\( F_{>4 \text{ MeV}} \)) and neglected (\( F_{>4 \text{ MeV}}^{\text{Gauss}} \)), and their ratio \( F_{>4 \text{ MeV}}^{\text{Gauss}}/F_{>4 \text{ MeV}} \), are exhibited in Fig. 6(b) for the case that \( T_e=10 \text{ keV} \). In this paper, as one of the typical case we choose the cutoff energy as 4 MeV. It should be noted that both \( F \) and \( F^{\text{Gauss}} \) parameters increase (\( F/F^{\text{Gauss}} \) value decreases) with decreasing cutoff energy. When \( T_e=10 \text{ keV} \), \( n_e=6.2 \times 10^{19} \text{ m}^{-3} \), \( P_{\text{NBI}}=20 \text{ MW} \) and \( E_{\text{NBI}}=1 \text{ MeV} \), \( F_{>4 \text{ MeV}}^{\text{Gauss}}/F_{>4 \text{ MeV}} = 1.9 \) (\( F_{>4 \text{ MeV}} = 3.8 \% \)), \( F_{>4 \text{ MeV}}^{\text{Gauss}}/F_{>4 \text{ MeV}} = 2.0 \% \) and \( F_{>3.9 \text{ MeV}}^{\text{Gauss}}/F_{>3.9 \text{ MeV}} = 1.5 \) (\( F_{>3.9 \text{ MeV}} = 7.1 \% \)), \( F_{>3.9 \text{ MeV}} = 4.8 \% \)). In this paper we have considered NBI and NES as mechanisms to create the non-Maxwellian tail component in deuteron distribution function. As the modification of emission spectrum, the effect of NES is small. When \( T_e = 20(10) \text{ keV} \), \( n_e=6.7(6.2) \times 10^{19} \text{ m}^{-3} \), \( P_{\text{NBI}}=20 \text{ MW} \), \( E_{\text{NBI}}=1 \text{ MeV} \), contribution of NES to the \( F_{>4 \text{ MeV}}^{\text{Gauss}}/F_{>4 \text{ MeV}} \) parameter is estimated to be less than 3(1)%.

The use of the \( \gamma \)-ray-generating \(^{9}\text{Be} (\alpha,\gamma)^{12}\text{C} \) reaction as a diagnostic tool has been proposed by Kiptily et al.\(^\text{17,18}\)
particular the energetic $\alpha$-particle can produce the $^{12}$C nuclei with the second excited state (7.65 MeV), and when the excited $^{12}$C transits to the first excited state (4.44 MeV), 3.21 MeV $\gamma$-ray is generated, i.e., 7.65–4.44 transition. The $\gamma$-rays (both 3.21 and 4.44 MeV) from the nuclear reaction between fusion-born $\alpha$-particle and $^9$Be impurity were observed in Joint European Torus (JET) “tritium” discharges. (In the experiment, the $\alpha$-particle emission spectrum would be regarded as almost Gaussian distribution.) The cross section for the 3.21 MeV $\gamma$-ray-generating $^{9}$Be($\alpha,n\gamma$)$^{12}$C reaction rapidly rises in the center-of-mass energy range above \~3.8 MeV, thus by looking at the 3.21 MeV $\gamma$-ray we could obtain information on $\alpha$-particle with energy above \~3.8 MeV. The 3.21 MeV $\gamma$-ray generation rate from $^{9}$Be($\alpha,n\gamma$)$^{12}$C reaction $Y_{\gamma(3.21)}$ is evaluated using the $\alpha$-particle velocity distribution function. We additionally carry out the BFP simulations assuming Gaussian distribution for $\alpha$-particle emission spectrum, and using the obtained $\alpha$-particle velocity distribution function the 3.21 MeV $\gamma$-ray generation rate (when $\alpha$-particle emission spectrum is assumed to be Gaussian distribution) $Y_{\gamma(3.21)}^\text{Gauss}$ is also evaluated. In the calculations the $^9$Be distribution function is assumed to be Maxwellian with the same temperature as bulk ion. Because of the large mass compared to deuteron, the distortion of $^9$Be distribution function would be negligible. The enhancement of the $\gamma$-ray generation rate due to modification of $\alpha$-particle emission spectrum $Y_{\gamma(3.21)}/Y_{\gamma(3.21)}^\text{Gauss}$ is shown in Fig. 7(a) as a function of NBI power for several electron temperatures. The calculation parameters except temperature are the same as those in Figs. 1–3. A significant enhancement of the $\gamma$-ray generation rate can be seen, especially in the low ($\leq 10$ keV) temperature range. The result shows the fraction of energetic ($>3.8$ MeV) $\alpha$-particles increases owing to the modification of their emission spectrum. By observing the enhancement of the 3.21 MeV $\gamma$-ray generation rate for various NBI powers and plasma conditions, we could sufficiently ascertain the appearance of the $\alpha$-particle born with energy larger than 3.8 MeV. The absolute values of the $\gamma$-ray generation rate from $^9$Be($\alpha,n\gamma$)$^{12}$C reaction when electron temperature is 10 keV are also presented in Fig. 7(b). In this case, for example, $^9$Be concentration is taken as $n_{^9\text{Be}}=0.005(n_D+n_T)$.

IV. CONCLUDING REMARKS

The $\alpha$-particle and neutron emission spectra in the presence of NBI heating have been evaluated and it has been shown that the fraction of energetic ($\geq 4.0$ MeV) $\alpha$-particle increases owing to the modification of the emission spectrum. When electron temperature $T_e=20(10)$ keV, electron density $n_e=6.7(6.2)\times 10^{19}$ m$^{-3}$, NBI power $P_{\text{NBI}}=40$ MW and energy $E_{\text{NBI}}=1$ MeV are assumed, the fraction of the power carried by $\alpha$-particle with energy above 4 MeV to total $\alpha$-particle power reaches 11.7% (5.1%), which is roughly 1.3 (2.4) times larger than the value when Gaussian distribution is assumed for the $\alpha$-particle emission spectrum, i.e., 9.1% (2.1%). In the same condition, the fraction of the power carried by neutron with energy above 14.5 MeV to total neutron power reaches 13.6% (6.1%), which is
influences of the broadening of the emission spectrum can be seen. By looking at the enhancement of the 3.21 MeV γ-ray generation rate (γ-ray emission spectrum) would be necessary.

The transport processes, i.e., confinement properties, of α-particle in the fusion devices tend to be affected by the α-particle’s energy. The increment in the fraction of energetic α-particle may influence the energy transfer processes from α-particle to background plasma. So far many studies to predict the α-heating characteristics have been made. The underlying physics model except NES in the present paper would be equivalent to the previous ones. In many previous simulations, however, monoenergetic fusion-born α-particle source has been assumed and the Fokker–Planck equation has not been solved considering the modification of the emission spectrum. By utilizing the enhancement of the α-particle spectrum. By looking at the enhancement of the 3.21 MeV γ-ray generation rate (γ-ray emission spectrum) would be necessary.

Especially in a low-temperature plasma, a significant enhancement of the 3.21 MeV γ-ray generation from 9Be(α, nγ)12C reaction due to modification of the α-particle emission spectrum can be seen. By looking at the enhancement of the γ-ray generation rate for various NBI conditions, we could ascertain the modification of the α-particle emission spectrum. By utilizing the enhancement of the γ-ray generation during NBI heating, a new approach to diagnose the energetic-ion velocity distribution function may be considered. More detailed analyses considering the correlation between the α-particle spectrum (distortion of fuel-ion velocity distribution function) and 3.21 MeV γ-ray generation rate (γ-ray emission spectrum) would be necessary.